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Nonlinear waves in Hertzian granular chains: Effects of inertial and stiffness heterogeneities

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Abstract: A one-dimensional granular medium, a chain of beads which interact via the nonlinear Hertz potential, exhibits strongly nonlinear behaviors. When an alignment of grains further contains some heterogeneities (e.g. local modification of the inertia or of the stiffness), it exhibits even more complex behaviors. We report some recent experiments, analysis and numerical simulations concerning nonlinear propagation of pulses in such alignments. We consider first a monodisperse chain as a reference case. We then analyze how pulse modifies in heterogeneous configurations. We show that strongly nonlinear behaviors take place if a chain contains a single intruder or if it contains a sharp decrease of bead's size. Finally, we present recent results concerning a monodisperse chain in which additional fluid is set between grains.

Keywords: one-dimensional granular chain, nonlinear phenomena, solitary wave, heterogeneous medium.

A. Introduction

The problem of mechanical energy transport in alignments of grains has attracted significant attention in recent years [1]-[7]. If the chain is held within boundaries, it is well established that sound waves may propagate and acoustic speed increases with imposed static stress. In the presence of loading, linear acoustic waves disperse as a function of time and space [8]. When the chain is not loaded, i.e., when the grains barely touch one another, acoustic speed vanishes: the medium cannot sustain linear acoustic waves. In this *sonic vacuum* limit, nonlinear phenomena predominate and any perturbation is known to result in strongly nonlinear waves which travel through the chain [1]. The spherical grains interact via the intrinsically nonlinear Hertz potential, in which the repulsive potential between the grains increases with grain-grain overlap via a 5/2 power law. This power law is softer than the quadratic (harmonic) repulsion for small overlaps but rapidly becomes steeper as compression increases. Hence, it is energetically expensive for two adjacent grains to sustain contact for long. This physical feature is at the heart of the peculiar nonlinear properties associated with mechanical energy transport in non-loaded granular alignments. It can be shown that whenever the repulsive potential has a steeper than quadratic power law growth in the overlap (or intergrain distance), mechanical energy transfer from one grain to the next starts off slowly and then ends abruptly. Energy is hence transported as a “bundle.” These alignments therefore propagate energy in the form of solitary waves. From a geometrical standpoint, the solitary wave in a granular alignment of spheres is a highly symmetric object whose velocity depends on the amount of energy it carries.

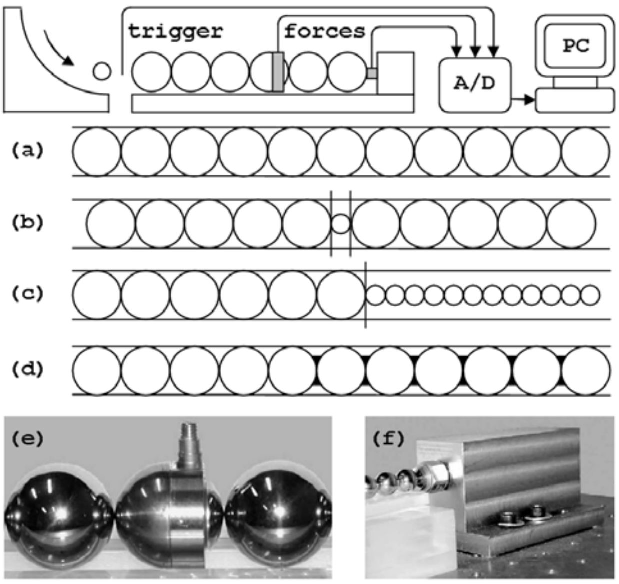

Fig.1. Experimental setup: Schematic view showing a chain with sensors and data acquisition facilities. The chain can be either (a) monodisperse, (b) may contains an intruder or (c) may contain a steep decrease in beads’ size. In configuration (d) we examine a wet monodisperse, in which interstitial viscous fluid has been added to some grains' contact. (e) Force sensor embedded in a bead and (f) rigid and non moving force sensor at the end of the chain.

In an ideal point of view, a clear understanding on how energy propagates in these alignments might be of interest to shed some light on real granular beds' dynamics at macroscopic scale. In a famous paper, Liu and Nagel [9] shown that sound propagation could be related to the existence of one-dimensional force networks in unconsolidated granular materials. However, Somfai and Roux [10] recently demonstrated that the link between sound, force networks and stiffness networks is definitively not obvious. In fact, single alignments do not

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take into account for essential complex nature of 3D granular materials. The transposition should thus be considered with care. Nevertheless, the grain-grain dynamics being relevant at the microscopic scale, whatever the macroscopic arrangement may be, this drawback may constitutes an asset. This ”lack” of complexity gives the opportunity to use these simple alignments as useful tools in probing some specific behaviors of granular materials that couldn't easily be identified in more general systems.

In this paper, we report recent experimental studies, analysis, and numerical simulations, devoted to understand how an initial perturbation provided to the edge of unconsolidated granular chains can result in the formation and the propagation of nonlinear waves, when the medium is heterogeneous. First, we report some experiments done in a monodisperse chain, as a reference configuration. Then, we analyze the effect of a local modification of chain's inertial properties, by adding a single intruder with smaller mass in a monodisperse chain. The following is devoted to the study of a stepped chain, a chain containing a sharp decrease of beads' size. Finally, we explore the effects of contacts' stiffness modification, by adding a small amount of viscous fluid between some grains of a monodisperse chain.

B. Experimental setup and nonlinear long-wavelength description

The setups under experimental study are one dimensional non-loaded chains of beads. All our beads are high carbon chrome hardened *AISI 52100* steel roll bearing (density $\rho = 7780\ \mathrm{kg/m^3}$, Young modulus $Y = 203\ \mathrm{GPa}$, and Poisson ratio $\upsilon = 0.3$), but their radii R and masses $m = (4/3)\pi\rho R^3$ differ from one chain to the other. The chain is impacted by a spherical striker initially moving at velocity V_S and whose radius is $R_S = 8\ \mathrm{mm}$. Typical elements of our experimental setups are shown in Fig.1. Measurements are achieved by using two piezoelectric force sensors. One of the sensors is boxed in a rigid and non moving heavy piece of metal placed at the end of the chain, and the second one is inserted inside a 13 mm radius bead that has been cut in two parts. The mass of the bead-sensor element has been rectified to match the mass of an original bead. The front mass being negligible, the embedded sensor provides a satisfactory estimation of the force at the contact with the neighbor bead [6].

An impact of the striker generates an elastic deformation of contact regions between beads, which propagates from one bead to another [1]-[2], [8]. The overlap at contact n (i.e. between beads n and $n-1$) is defined as $\delta_n = (u_{n-1} - u_n)$, where u_n is the position of bead number n. Derived from Hertz potential [11]-[12], the force at contact n is $F_n = \kappa\delta_n^{3/2}$ if $\delta_n > 0$ (and 0 if $\delta_n \le 0$), where $\kappa = R^{1/2}/(2^{3/2}\theta)$ and $\theta = 3(1-\upsilon^2)/4Y$ are constants. More precisely, the dynamics of the chain is described by applying Newton's second law to each of the beads,

$$m\ddot{u}_n = \kappa(u_{n-1} - u_n)^{3/2} - \kappa(u_n - u_{n+1})^{3/2}. \qquad (1)$$

A continuum limit of this set of equations can be derived under the long wavelength approximation, when characteristic wavelength λ of the deformation is greater than the radii of the beads [8]. Keeping terms of up to the fourth order of small parameter $(2R/\lambda)$ in Taylor expansions of $u_{n\pm1} = u(x_n \pm 2R, t)$ versus $u_n = u(x_n, t)$ leads to a nonlinear equation for the strain $\psi(x,t) = -\partial u(x,t)/\partial x$,

$$\ddot{\psi} = (2R)^{5/2}\left(\frac{\kappa}{m}\right)\left[\psi^{3/2} + \left(\frac{2}{5}\right)R^2\psi^{1/4}\left(\psi^{5/4}\right)_{xx}\right]_{xx}. \qquad (2)$$

An exact solution of the last equation is,

$$\psi = \psi_m \cos\xi,\ \xi = \frac{x - vt}{R\sqrt{10}},\ v = \sqrt{\frac{6}{5\pi\rho\theta}}\psi_m^{1/4}, \qquad (3)$$

where one hump of the strain wave ψ, for $|\xi| \le \pi/2$, describes a single solitary wave (SW) solution [1]-[2], [8], with a compact spatial extent of the order of the characteristic length $\lambda_{SW} = R\sqrt{10}$. The duration of the pulse is approximately two times the amplitude dependent characteristic duration $\tau_{SW} = R\sqrt{10}/v$.

In addition to theoretical analysis, the discrete set of equations (1) is solved numerically by using a fourth order Runge-Kutta numerical scheme, as proposed by Chatterjee [13]. The time step is chosen to be few orders of magnitude smaller than the shortest physical duration and the energy conservation is fulfilled within an error less than 10^{-9}.

C. Dry chain of identical beads

We present here some experimental observations carried out in a non-loaded monodisperse chain [6]. The chain, shown in inset (a) of Fig.1, is made of identical beads with radius is $R = 13\ \mathrm{mm}$. As previously demonstrated experimentally and numerically [4], [8], [13], the initial impact being applied by a light striker ($R_S = 8\ \mathrm{mm}$), a single solitary wave propagates in the chain. Such a wave is shown in inset (a) of Fig.2, which represents the force measured as it propagates deeper in the chain by the bead/sensor element. The pulse slightly changes while propagating the distance of the few first beads, and then quickly reaches a stable profile. It should be pointed out that previous experimental results [4], mainly focusing on loaded monodisperse chains, didn't reach sufficiently precise measurements in the non-loaded limit to allow a satisfactory comparison with theory, (3). Inset (b) and (c) of Fig.2 show the wave velocity and the wave duration respectively as a function of the incident wave force amplitude. In our experiments, the solitary wave profile, its velocity and its duration exhibit excellent agreements with theoretical results of (3), without any adjustable parameter.

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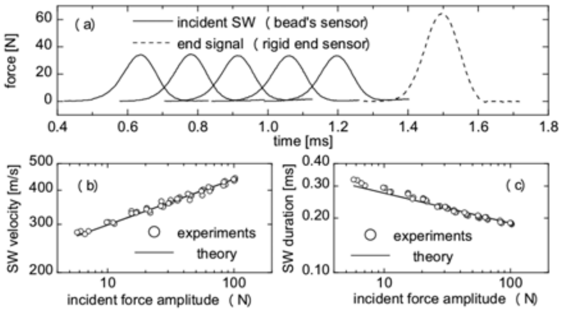

Fig.2. Experiments in a non-loaded monodisperse chain. (a) Incident force at different position in the chain, and force measured at the rigid end, as a function of time. (b-c) Solitary wave velocity and duration, respectively, as functions of incident force amplitude. Theory estimates are based on (3).

D. Effects of an intruder

The simplest way to explore the role of heterogeneities is to insert a single intruder in a monodisperse chain [14]-[15]. Such a configuration is shown in inset (b) of Fig.1. A qualitative description of the chain's dynamics can be achieved, provided the monodisperse chain of beads can be seen as a discrete chain of identical mass m connected by linear springs with stiffness $k = \partial F / \partial \delta$. According to linear theory, the dispersion relation indicates that acoustic wave can propagate in such a linearized chain if the angular frequency of the perturbation is lower than a given cutoff frequency, $\omega_C \approx \sqrt{k/m}$. Any faster perturbation will result in oscillatory non-propagating evanescent waves. We consider now the presence of a light intruder with mass m_I. The natural angular frequency of the light intruder, $\omega_I \approx \sqrt{k/m_I}$, may exceed the cutoff angular frequency. A localized state thus may appear. The intruder starts to oscillate from any dynamical perturbation and these oscillations remains localized on the intruder as they cannot propagate in the chain. This phenomenon can be observed experimentally and numerically, as shown in Fig.3. In the force measured few beads before the intruder, the first pulse is the incident wave, and the second smaller one is a reflected wave. The reflected pulse indicates that, from energy and momentum conservations considerations, a gap opened in the chain, in the close vicinity of the intruder. In both figures, the oscillating tail in the force measured at the intruder contact is a clear signature of the oscillation of the intruder. We checked that these oscillations does not propagate in the chain, they thus remain localized on the intruder. The excellent agreement between experiments and simulations demonstrates the reliability of our measurements. According to experimental results and considering a more detailed analysis, which will be published elsewhere [15], the angular frequency of these oscillations are found to depend on both the incident wave features and the geometrical properties of the chain,

$$\omega_I \propto \left(\frac{R}{R_I}\right)^{4/3} F_m^{1/6}, \qquad (4)$$

where R and $R_I << R$ are radii of the beads and of the intruder, respectively, and F_m is the incident force amplitude.

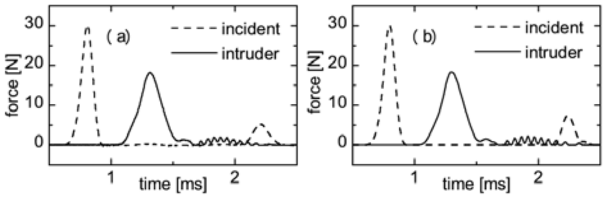

Fig.3. Monodisperse chain containing an intruder. (a) Experiments. (b) Numerical simulation. The incident wave is measured few beads before the intruder. The oscillation in the tail of the force at the intruder is the clear signature of wave localization on the intruder.

E. Effects of a step in beads's size

Attention now turns to mechanical energy propagation due to initial perturbations that generate significantly long grain compressions at a chain end. Lazaridi and Nesterenko demonstrated that an initial perturbation provided at the edge of a non-loaded granular chain by a heavy striker can result in the formation of solitary wave trains, containing many single solitary waves ordered by decreasing amplitudes [2]. Solitary wave trains are also known to exist in stepped chains of beads [16]-[17], i.e., chains containing a steep decrease in bead radius as shown in inset (c) of Fig.1. A stepped chain is similar, from the point of view of the interaction of the last bead of the first chain with edge beads of the second smaller chain, to a monodisperse chain impacted by a large and massive striker [2], [16]-[18]. Interestingly, Nesterenko reported that energy and momentum can be totally transmitted when a pulse passes through a sharp decrease of bead's size, and partial reflected in the reverse case [16]-[17]. Such nontrivial and fascinating behaviors are highly related to the way energy and momentum transfer nonlinearly at the interface between chains. This type of interaction has practical interests, since they are known to act as efficient shock absorbers [19]. Solitary wave trains formation observations and description have been confirmed and extended in recent works [20]-[21]. The force amplitude of the p^{th} single pulse in a solitary wave train has been found to decrease approximately exponentially, i.e. $F_m \propto \exp(-\alpha p)$. Applying energy and momentum conservations at the interface between both parts of the stepped chain, we found that the coefficient α only depends on the beads' mass of the first and the second part of the chain, respectively M and m,

$$\alpha = \left(\frac{6}{5}\right) \ln\left[\frac{1+(m/M)\Omega}{1-(m/M)\Omega}\right], \qquad (5)$$

where $\Omega \approx 1.345$ is a non-dimensional constant coming from theory [21]. We performed measurements in a stepped chain, which are presented in Fig.4. The theoretical estimate we proposed is found in a good agreement with both numerical simulations [20] and experiments [21]. Moreover, we shown that the force felt at the interface, measured by the embedded sensor first loads within a characteristic time of the order of incident solitary wave duration, and then unloads through a longer exponential relaxation-like process. We found [21] that the characteristic unloading time is inversely proportional to the coefficient α. Finally, we demonstrated that the

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overall strength of the transmitted solitary wave train decreases when the mass ratio M/m increases, whereas the number of pulses it contains increases. This last result proves valuable in using such systems as shocks protectors.

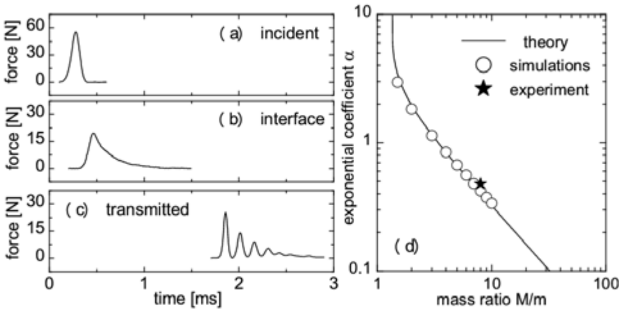

Fig.4. Stepped chain made with 7 beads of radius $R_1 = 13$ mm followed by 50 beads of radius $R_2 = 6.5$ mm. (a) Incident solitary wave's force as a function of time in the first chain. (b) Force at interface versus time. (c) Solitary wave train's force measured in the second part of the chain. (d) Coefficient α versus beads' mass ratio, defined such that $F_m \propto \exp(-\alpha p)$ where $F_m(p)$ is the maximum force of the p^{th} transmitted solitary wave.

F. Effects of an interstitial fluid

The influence of interstitial fluid on grains' interaction still suffers a lack of understanding despite major practical importance in describing wet granular media dynamics. In recent ultrasounds propagation experiments, Jia and Mills [22]-[23] reported that the coherent ballistic wave and the coda wave both dissipate if some viscous fluid is mixed with the grains. Interestingly, they also reported that the velocity of the ballistic wave can increase, by up to 25%, even if the volume fraction of fluid was very small, about 0.06% in their experiments. Their interpretation of such stiffening effect, based on an effective medium description, assumes the presence of liquid bridges between grains in parallel to dry Hertzian contacts. Characterizing these liquid bridges by simple Maxwell elements (a dashpot and a linear spring in series), they found qualitative agreements with experiments that might explain the pulse acceleration.

Some of these effects have also been observed in beads alignments. Herbold and Nesterenko reported experiments done in a monodisperse chain of beads immersed in glycerol fluid (dynamic viscosity $\eta = 0.62$ N.s/m^2) [24], where slight acceleration of the pulse was underlined. Performing numerical simulations, they included classical buoyancy, drag and pressure forces. In addition to these effects, they also proposed to take into account for a viscous term in the set of equations (1), in the form,

$$m\ddot{u}_n = \kappa\left(\delta_n^{3/2} - \delta_{n+1}^{3/2}\right) + \mu\left(\dot{\delta}_n - \dot{\delta}_{n+1}\right), \qquad (6)$$

where μ is a viscous coefficient proportional to the dynamic viscosity. It is noteworthy that the model exposed in (6) should lead to an increase of the velocity, when considering a short enough pulse, i.e. a wave with sufficiently high frequency content. Their numerical simulations qualitatively shown that the dissipation of the pulse can be fitted by appropriate selection of μ, but they also found a wave deceleration, in contradiction to experiments.

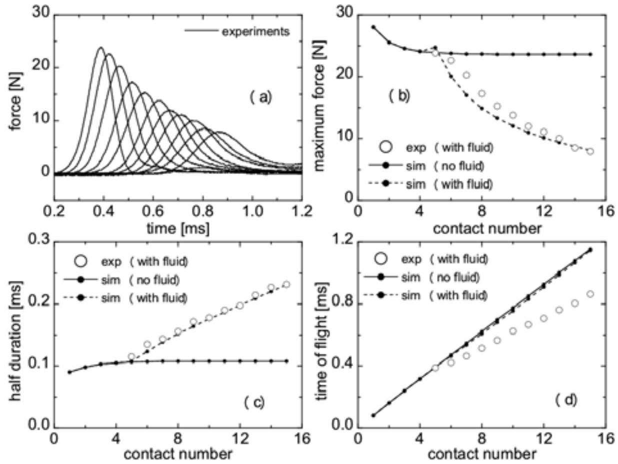

Fig.5. Monodisperse chain with additional interstitial viscous fluid set at contacts 6 to 15. Numerical simulation, performed by setting viscous coefficient to $\mu = 250$ N.s/m, shows satisfactory agreements for the pulse's amplitude decrease and broadening, but fails to describe the pulse acceleration. Without fluid, the pulse velocity is 341 m/s, and in the presence of interstitial fluid, the velocity increases by 56%, up to 531 m/s.

We performed measurements in a monodisperse chain, in order to test Jia's and Mills's interpretations. The configuration is shown in inset (d) of Fig.1. Contrarily to Herbold's approach, our chain is not immersed, in order to avoid any additional effects, and a very small amount of viscous fluid is set between grains. The fluid we use, Sylgard 184, is a highly viscous one ($\eta = 5.5$ N.s/m^2), in order to ensure a low enough Reynolds number. Measurements are shown in Fig.5, where pulse dissipation and acceleration are clearly demonstrated. Performing numerical simulations of the discrete set of equations (6), we found that setting the viscosity coefficient to $\mu = 250$ N.s/m gives a satisfactory agreement for the amplitude decreasing and pulse broadening. The pulse acceleration is also observed in our numerical simulations, but does not agree with experimental observations. Our measurements actually confirm Herbold's assumption upon the introduction of a viscous term proportional to the relative velocity [24]. Indeed, grains' overlaps might generate a mean radial flow proportional to relative grains' velocities. In order to get insight of the microscopic scale, we try in the following to relate the numerical viscous coefficient μ to the physical properties of the setup.

In the flied of Tribology [25], it is known that a squeezing effect, a classical elastohydrodynamic lubrication effect [26], generally takes place at low Reynolds number when two solids, separated by a thin layer of fluid, are pressed together. It is well established for example that when two circular plates (radius R), separated by a viscous fluid (dynamic viscosity η and thickness h), are approached with relative velocity V_R, a normal load opposed to the displacement is induced, $F = \mu V_R$, where $\mu \sim R^4/h^3$. Similarly, if the solids are two rigid spheres with radius R, separated by distance h, the coefficient scales as $\mu \sim R^2/h$. Assuming a film

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of liquid should form in the vicinity of the grains' contact region, we find from the geometrical properties of the setup, and from the numerically found viscosity coefficient $\mu = 250$ N.s/m, that this film should be few microns thick. Considering the overlap is also of the order of few microns, the interpretation of a squeezing flow thus appears relevant.

At this stage, the stiffening effect was not unraveled. Following Jia's and Mills' proposal [22], the pulse acceleration should take place by considering additional restoring forces, e.g., by considering fluid compressibility with appropriate description.

G. Conclusion

In summary, we have reported detailed studies of the propagation of strongly nonlinear wave in various non-loaded chains of beads. We first presented some results done in a monodisperse chain. These results confirmed the reliability of existing theoretical results, and showed that a pulse propagates in the form of a solitary wave. We then analyzed the influence of a single intruder in a monodisperse chain, as the simplest heterogeneous configuration. A well defined nonlinear wave localization process occurs at the location of the intruder. Further, we explored the propagation and the transmission of solitary wave in a stepped chain and we reported recent analysis that allowed describing the main features of a transmitted solitary wave train. Finally, we exposed recent experiments and interpretation concerning the propagation of a wave in a Hertzian chain with additional interstitial fluid. The physical description of such a system remains opened, but dissipative and stiffening effects were clearly shown experimentally. All these observations should prove valuable to reach a better knowledge of the dynamics of more complex granular material.

H. Acknowledgements

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